

 | IME IN PRIIMEK: _____

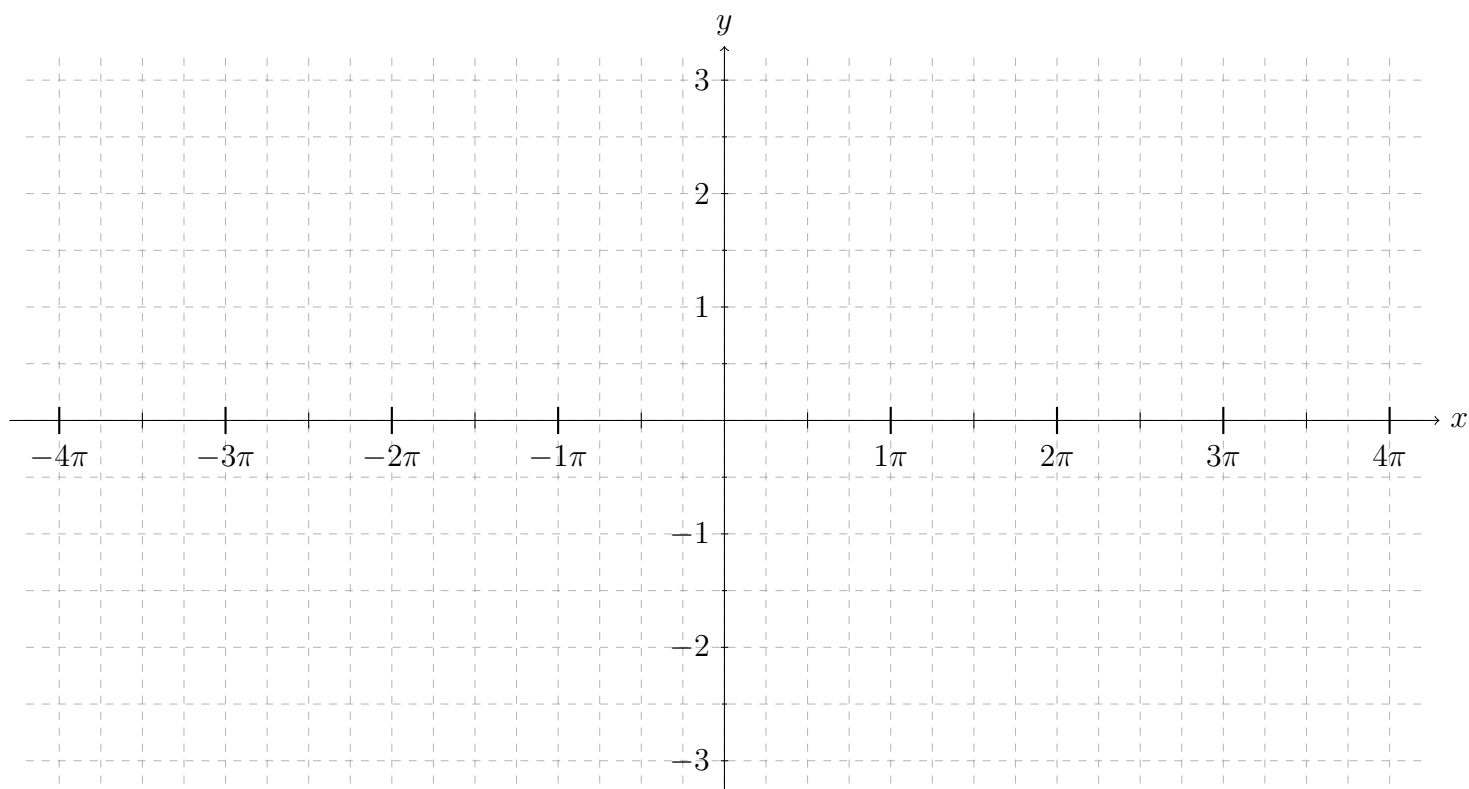
Naloga 1:	3 + 4 + 4	↔ a.	b.	c.
Reši enačbo:				
a.) $2 \cos(x + \pi) = -1$				
Rešitev: _____				
b.) $\tan^2 x = \tan x + 2$				
Rešitev: _____				
c.) $\sin 5x = \sin 3x$				
Rešitev: _____				

Podana je funkcija $f(x) = -3 \sin\left(\frac{1}{2}x + \frac{\pi}{4}\right)$.

b) Izračunaj ničle, zalogo vrednosti, začetno vrednost in periodo funkcije.

b) Določi $A\left(\frac{5\pi}{6}, y\right)$, da bo na grafu.

c) Nariši graf funkcije in na grafu označi maksimume in minimume.



Naloga 3:

3 + 4

↔ |a.

|b.

|c.

|d.

|

Izračunaj:

a) $\arccos\left(\sin\left(\frac{\pi}{2} + \arctan\left(2 \sin \frac{\pi}{3}\right)\right)\right)$

b) s prehodom na ostri kot: $\frac{\sin^2 \frac{5\pi}{3} \cdot \cos\left(-\frac{7\pi}{3}\right)}{\cot\left(-\frac{3\pi}{4}\right) - \tan^2 \frac{2\pi}{3}}$

Naloga 4:

4

↔ |a.

|

Poenostavi: $\left(\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 - \cos x}\right) \cdot \sqrt{1 - \cot^2 x}$

Naloga 5:

3 + 3 + 3

↔ |a.

|b.

|c.

|d.

|

Naj bo $\cos \alpha = \frac{12}{13}$ in $\pi < \alpha < 2\pi$.

Izračunaj:

a) $\sin\left(\alpha + \frac{\pi}{4}\right)$

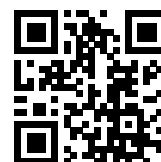
b) $\tan 2\alpha$

c) $\cos\left(2\alpha + \frac{\pi}{3}\right) - \cos\left(2\alpha - \frac{\pi}{3}\right)$

Število doseženih točk na testu:

število vseh točk na testu: 40

ocena	1	2	3	4	5	uspešnost v %	OCENA
%	[0, 45)	[45, 60)	[60, 75)	[75, 90)	[90, 100]		



Rešitve

Naloga 1

a)

$$2 \cos(x + \pi) = -1$$

$$\cos(x + \pi) = -\frac{1}{2}$$

$$x + \pi = \pm \frac{2\pi}{3} + 2k\pi$$

$$x = -\frac{\pi}{3} + 2k\pi \quad \text{ali} \quad x = -\frac{5\pi}{3} + 2k\pi, \quad k \in \mathbb{Z}$$

b)

$$\tan^2 x = \tan x + 2$$

$$\tan^2 x - \tan x - 2 = 0$$

$$(\tan x - 2)(\tan x + 1) = 0$$

$$\tan x = 2 \quad \text{ali} \quad \tan x = -1$$

$$x = \arctan 2 + k\pi \quad \text{ali} \quad x = -\frac{\pi}{4} + k\pi, \quad k \in \mathbb{Z}$$

c)

$$\sin 5x = \sin 3x$$

$$\sin 5x - \sin 3x = 0$$

$$2 \cos 4x \sin x = 0$$

$$\sin x = 0 \quad \text{ali} \quad \cos 4x = 0$$

$$x = k\pi \quad \text{ali} \quad x = \frac{\pi}{8} + \frac{k\pi}{4}, \quad k \in \mathbb{Z}$$

Naloga 2

Dana je funkcija

$$f(x) = -3 \sin\left(\frac{1}{2}x + \frac{\pi}{4}\right).$$

Ničle:

$$\sin\left(\frac{1}{2}x + \frac{\pi}{4}\right) = 0$$

$$\frac{1}{2}x + \frac{\pi}{4} = k\pi$$

$$x = 2k\pi - \frac{\pi}{2}, \quad k \in \mathbb{Z}$$

Zaloga vrednosti:

$$[-3, 3]$$

Začetna vrednost:

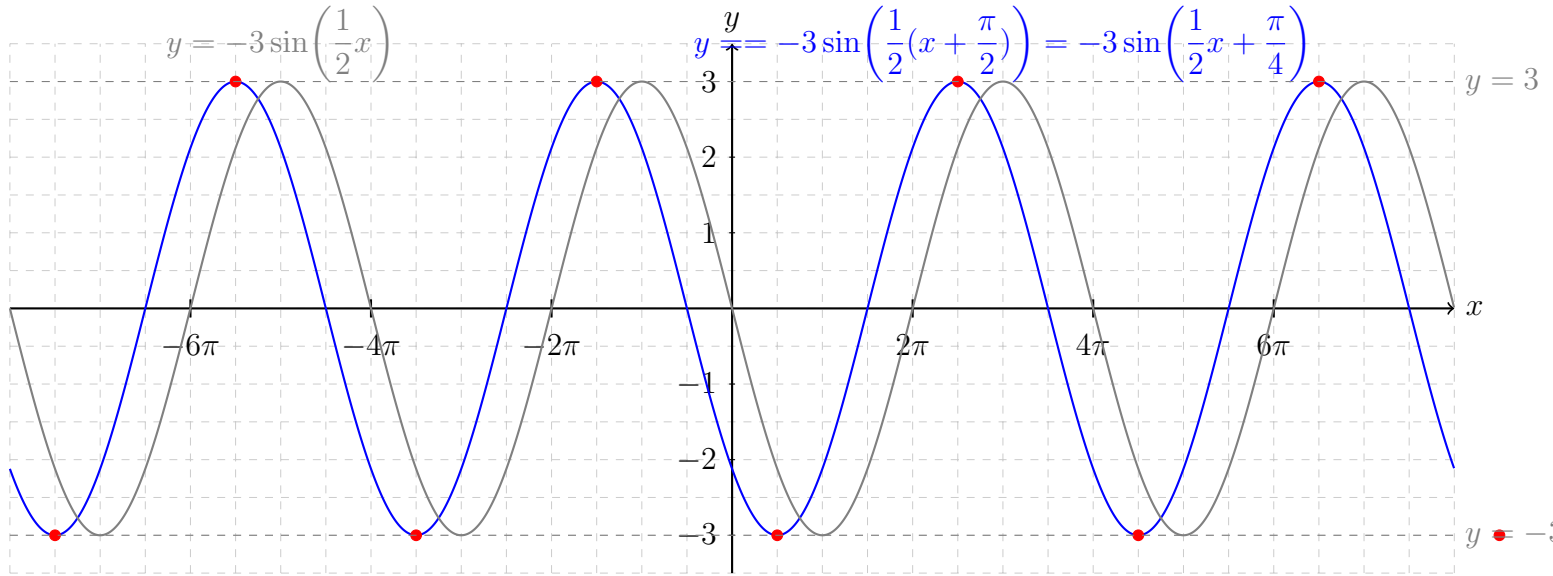
$$f(0) = -3 \sin \frac{\pi}{4} = -\frac{3\sqrt{2}}{2}$$

Perioda:

$$T = \frac{2\pi}{1/2} = 4\pi$$

Točka $A\left(\frac{5\pi}{6}, y\right)$:

$$y = -3 \sin \left(\frac{5\pi}{12} + \frac{\pi}{4} \right) = -3 \sin \frac{2\pi}{3} = -\frac{3\sqrt{3}}{2}$$



Naloga 3

a)

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow 2 \sin \frac{\pi}{3} = \sqrt{3}$$

$$\arctan(\sqrt{3}) = \frac{\pi}{3}$$

$$\arccos\left(\sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right)\right) = \arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$$

b)

$$\sin^2 \frac{5\pi}{3} = \frac{3}{4}, \quad \cos\left(-\frac{7\pi}{3}\right) = \frac{1}{2}$$

$$\cot\left(-\frac{3\pi}{4}\right) = 1, \quad \tan^2 \frac{2\pi}{3} = 3$$

$$\frac{\frac{3}{4} \cdot \frac{1}{2}}{1 - 3} = -\frac{3}{16}$$

Naloga 4

$$\begin{aligned} & \left(\frac{1 + \cos x}{\sin x} + \frac{\sin x}{1 - \cos x} \right) \sqrt{1 - \cot^2 x} \\ &= \frac{2}{\sin x} \sqrt{\frac{\sin^2 x - \cos^2 x}{\sin^2 x}} = \frac{2}{\sin x} \cdot \frac{|\sin^2 x - \cos^2 x|^{1/2}}{|\sin x|} \end{aligned}$$

(poenostavitev je odvisna od definicijskega območja)

Naloga 5

Ker je $\pi < \alpha < 2\pi$ in

$$\cos \alpha = \frac{12}{13},$$

sledi

$$\sin \alpha = -\frac{5}{13}, \quad \tan \alpha = -\frac{5}{12}.$$

a)

$$\sin \left(\alpha + \frac{\pi}{4} \right) = \sin \alpha \cos \frac{\pi}{4} + \cos \alpha \sin \frac{\pi}{4} = \frac{7\sqrt{2}}{26}$$

b)

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha} = -\frac{120}{119}$$

c)

$$\cos(2\alpha + \frac{\pi}{3}) - \cos(2\alpha - \frac{\pi}{3}) = -2 \sin 2\alpha \sin \frac{\pi}{3} = \frac{120\sqrt{3}}{169}$$